DIFFUSION OF GAS COMPONENTS IN TWO-PHASE SYSTEMS WITH VIBRATIONAL PERTURBATION

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A mathematical model is proposed for the liberation of the gas component in two-phase systems with vibrational perturbation, and an algorithm for the numerical integration of this model is presented.

In [1-6], the results of investigations of the phase interaction in nonsteady fluxes of slightly concentrated suspensions of small spheres were presented. An earlier work [7] demonstrated the laws governing the propagation of harmonic oscillations in two-phase systems, where the relative velocity of air-bubble motion was taken from the Stokes formula. A two-phase system in which one component is a liquid and the other a gas was modeled in the form of an elastic–viscous continuous medium with air bubbles being liberated under vibration.

In the present work, an attempt is made to develop a mathematical model of the diffusion of the gas component from two-phase systems in the atmosphere under asymmetric vibrational perturbations. The use of asymmetric (e.g., vibroshock, multifrequency) oscillations allows the technological reprocessing of disperse materials to be sharply intensified [8, 9]. Change in the particle concentration of the gas phase under vibrations of the medium may be represented in the form of the Einstein–Kolmogorov equations [10]. It is assumed that the Reynolds number is small, and the bubbles are spherical in form, while the change in pressure in the liquid phase due to bubble motion does not change its shape, i.e., its diameter remains constant. In addition, it is assumed that the boundary layer does not break away from the bubble surface, and the gas motion inside the bubble has no effect on the motion of the liquid phase [11].

The equation of motion of a bubble in a viscous medium takes the form

$$\rho_T V \frac{dv}{dt} = -\rho_T g V + (g+a)\rho V - 3\pi d\mu (v-u) - K_{ad}\rho V \frac{d(v-u)}{dt}, \tag{1}$$

where K_{ad} is the added-mass factor ($K_{ad} = 0.5$) [12, 13].

Since $\rho_T/\rho \approx 0$, denoting the relative velocity of the vibrated air bubble by $v_b = v - u$, Eq. (1) takes the form

$$K_{ad}\frac{dv_b}{dt} + \xi(w) v_b = g + a, \tag{2}$$

where $\xi = 18\nu/(1-w)d^2$ is the drag coefficient of the two-phase medium.

The mathematical model of the gas component of the central axisymmetric layer for a plane problem takes the form

$$\frac{\partial w}{\partial t} = -v_b \frac{\partial w}{\partial x} + D\frac{\partial^2 w}{\partial x^2}, \qquad K_{ad}\frac{dv_b}{dt} + \xi(w) v_b = g + a, \tag{3}$$

where the relative velocity of the air bubble depends on the particle concentration of the gas component, while the dependence of the diffusion coefficient on the parameters of the vibrational perturbation and the rheological properties of the medium is determined by means of numerical experiment.

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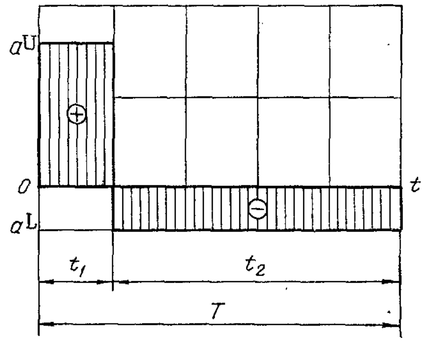

Fig. 1

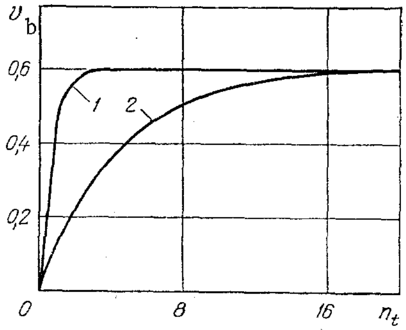

Fig. 2

Fig. 1. Change in acceleration of vibrating plane after the period of the asymmetric vibrations.

Fig. 2. Dependence of velocity of air-bubble motion on the number of integration steps over the time: 1) ξ = 100 sec^{-1}; $K_{ad}K_T\omega$ = 100 sec^{-1}; 2) ξ = 100 sec^{-1}; $K_{ad}K_T\omega$ = 2400 sec^{-1}.

The vibrations of the perturbation source are governed by the law shown in Fig. 1, where t_1 is the time of impact of the vibrating plane on the stop; t_2 is the time that the plane returns to the initial position and $t_1 = t_2 = T = 2\pi/\omega$; T is the period of the vibration; ω is the angular velocity of the vibration. In the present case, the upper limit is the free surface. Assuming that initially the velocity of air-bubble motion is zero over the entire height of the two-phase system, and also specifying the law of initial distribution of the bulk particle concentration of the gas phase, the drag coefficient may be determined for a given moment of time. Taking account of the value of ξ obtained from the second relation in Eq. (3), the velocity v_b of air-bubble motion for a new moment of time, depending on the coordinate x and the time t, may be determined. The value of v_b obtained is used in the first relation of Eq. (3) to determine the change in bulk concentration of air associated with the change in the velocity of bubble motion.

Thus, the two relations in Eq. (3) are integrated together, step by step in terms of the coordinate and the time, until the specified residual concentration is obtained. The finite-difference analog of the system in Eq. (3) takes the form

$$w_i^{t+\delta t} = w_i p_{0i} + w_{i+1} p_{i+1} + w_{i-1} q_{i-1}, \tag{4}$$

$$v_{ni}^{t+\delta t} = \frac{\frac{\delta t}{K_{ad}}(g+a) + v_{ni}}{1 + \frac{\xi \delta t}{K_{ad}}},$$

where $p_{0i} = 1 - 2F$ is the probability of motion of a particle of the gas component (concentration) from the i-th point for the given moment to the same point for the new moment; $p_{i+1} = F - G_{i+1}$ is the probability of motion of a particle of the gas component from the (i + 1)-th point for the given moment to the i-th point for the new moment; $q_{i-1} = F + G_{i-1}$ is the probability of motion of a particle of the gas component from the (i − 1)-th point for the given moment to the i-th point for the new moment; $F = D\delta t/\delta x^2$; $G_i = v_{ni}\delta t/2\delta x$; $\delta t = 2\pi/\omega K_T$; K_T is the number of divisions in the oscillation period; δx is the coordinate step, $\delta x = H/(K_{x-1})$; K_x is the number of points of the integration grid over the column height H. Note that the concentration is defined in accordance with the first relation in Eq. (4) for all points over the height except the lowest point corresponding to the bottom of the vibrated volume.

The concentration at the lowest point of the integration grid may be defined by the formula

$$w_i^{t+\delta t} = w_i (p_{0i} + p_i) + w_{i+1} p_{i+1}, \tag{5}$$

since the particles of the gas component do not penetrate the bottom of the volume. The method of defining the particle concentration of the gas component is based on the theory of Markov processes [14], where $p_{0i} + p_i + q_i = 1$.

However, it should be noted that $p_{0i} + p_{i+1} + q_{i-1} \neq 1$, because of the dependence of the velocity of air-bubble motion on the column height (coordinate x) and the time. Since the theory of Markov processes entails that

$$p_{0i} = 1 - 2F \geqslant 0, \quad F \leqslant \frac{1}{2},$$

$$p_i = F - G_i \geqslant 0, \quad F \geqslant G_i,$$

$$q_i = F + G_i \geqslant 0, \quad F \geqslant -G_i$$

and bearing in mind that in the course of vibration the particles of the gas component are displaced upward, $q_i > p_i$, i.e., the probability of upward particle motion is higher than that for downward motion, it is found that

$$\frac{1}{2} \geqslant F \geqslant |G_i|. \tag{6}$$

It follows from this expression that

$$\delta t \leqslant \frac{\delta x^2}{2D} \tag{7}$$

and

$$|v_{ni}| \leqslant \frac{2D}{\delta x}. \tag{8}$$

The constraints in Eqs. (7) and (8) are necessary conditions for stability of the scheme of numerical integration of the given mathematical model, and indicate that the change in particle concentration of the gas component at points of the numerical scheme cannot be negative. The distinguishing feature of asymmetric vibrations, when the upper and lower accelerations of the vibrating floor may differ by (8-10)g, is that there is a change in the density of the two-phase system, associated not only with diffusion of the gas component but also with the appearance of considerable dynamic pressure in the vibrating medium. In addition, bubble motion in the given case is associated with a transient process whose duration depends on the drag of the medium and the parameters of the vibrational perturbation. Note also that, for a harmonic law of vibration of the bottom plane of the vibrated volume, $p_{imax} = q_{imax}$ and $p_{imin} = q_{imin}$. These conditions are satisfied for amplitude values of the acceleration that are equal in magnitude but opposite in sign. In the case of asymmetric vibrations, $p_{imax} \neq q_{imax}$ because of the difference in upper and lower accelerations of the vibrations of the bottom plane.

The dependence of the velocity of air-bubble motion on the number of integration steps over the time n_t is shown in Fig. 2 for different numbers of divisions of the oscillation period $K_T = T/\delta t = 2\pi/\omega\delta t$, with $\xi = 100$ sec^{-1} and upper and lower accelerations $a^U = 5g$ and $a^L = g$ of the vibrating bottom. Analysis of the investigations shows that the time of the transient process depends on the parameters of the vibrational perturbation and the rheological properties of the medium. Note that with increase in the drag coefficient of the medium to $\xi = 2 \cdot 10^4$ sec^{-1} and above, the duration of the transient process has no significant effect on the diffusion of the gas component.

It is of interest to study the grouping of the air-bubble flux and the motion of its center of gravity in the vibration of two-phase media. Initially, with uniform distribution of air bubbles, the center of gravity (CG) is in the central part of the volume's height. With vibration, the position of the center of gravity shifts, because of changes in the particle concentration of the gas component over the height. The results obtained are confirmed by the experimental investigations reported in [15, 16]. Investigation of the laws of motion of the center of gravity permits the vibration parameters and rheological parameters of the medium to be optimized so as to obtain the required density of the reprocessed two-phase system. Investigation shows that the center of gravity will be displaced upward under the condition $q_i - p_i \gtrsim 0.04$.

The dependence of the distribution of the bulk particle concentration of the gas component on the number of points (K_x) over the column height is shown in Fig. 3. Analysis of the data obtained shows that when the center of gravity moves upward there is more intense

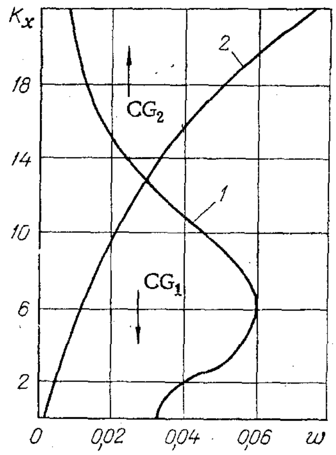

Fig. 3. Dependence of the distribution of the bulk particle concentration of the gas component on the number of points over the column height: 1) center of gravity (CG_1) falls; 2) CG_2 rises.

diffusion of the gas-component particles to the free surface (curve 2) with a monotonic increase in concentration from the vibrated plane upwards. When the center of gravity moves downward, the duration of the diffusion of gas-component particles increases considerably (curve 1), which leads to insufficient density in the vibrated medium. Since liberation of the gas component occurs as a result of diffusion, and also motion of air bubbles under vibration, the center of gravity rises in the case where the velocity flux predominates over diffusion and falls in the opposite case.

Investiation shows that the velocity of air-bubble motion depends on the rheological properties of the medium (ν, d, ρ, D), the column height (H), and the vibration parameters (a^U, a^L, ω). The upper acceleration (a^U) has the largest effect on the liberation of the gas component, which occurs most intensely in the interval t_1 (Fig. 1), when $a^U \approx (3\text{-}4)g$ and $a^L = g$. This is because the impact time is optimal in the selected range of change in acceleration. The velocity of bubble motion begins to influence the process of air liberation when $\nu/d^2 \leqslant 10\ \text{sec}^{-1}$, where the drag coefficient of the medium is slight. In the opposite case, the main influence on the liberation of the gas component is that of diffusion, and the velocity of bubble motion is negligibly small.

The dependence of the duration of gas-component liberation on the ratio ν/d^2, the angular velocity, the column height, and the diffusion coefficient has also been investigated. The time of liberation of the gas component was determined by numerical experiment on a computer when 50% of the gas phase had passed through the gas phase. The range of variation in the various factors was as follows: $\nu/d^2 = 1\text{-}201\ \text{sec}^{-1}$, $\omega = 20\text{-}100\ \text{sec}^{-1}$; H = 0.2-1.4 m; $D = 10^{-3}\text{-}10^{-1}\ \text{m}^2/\text{sec}$. In determining the approximating polynomial, a symmetric plan of the numerical experiment was used for four factors at five integer levels (−2; −1; 0; 1; 2) [17]; the independent variables involved were expressed as dimensionless quantities normalized over the levels, and the function t being determined was expressed in seconds.

Investigation shows that the time dependence of the quantity of gas component liberated is exponential in character, taking the form 1 − exp (−pt), where p is some factor. Therefore, the value of the time obtained in the numerical experiment performed will be three times smaller than the total time for the liberation of the gas component, to an accuracy of ±10%, which is acceptable for practical purposes. Approximating the dependence $t = f(\nu/d^2, \omega, H, D)$ by the least-squares method in the form of a complete quadratic gives

$$t = 18.5634 + 5.1276\,\overline{(\nu/d^2)} - 1.8524\,\overline{(\nu/d^2)}^2 - 1.9445\overline{\omega} + 5.7012\overline{H} + \\ + 2.5819\,\overline{(\nu/d^2)}\,\overline{H} - 8.2569\overline{D} - 2.9000\,\overline{(\nu/d^2)}\,\overline{D} - 2.4242\overline{H}\overline{D}, \tag{9}$$

where the quantities with bars are dimensionless normalized parameters, and the missing terms of the complete four-factor quadratic are negligibly small, and may be disregarded. Analysis of the approximating polynomial obtained shows that the strongest influence on the liberation of the gas component in the given region of factor space comes from diffusion, followed by the column height and the ratio of the kinematic viscosity of the medium to the square of the air-bubble diameter. The dispersion here is 673.516, the mean-square deviation is 6.488, and the correlational ratio is 0.945, which is acceptable for the practical use of the polynomial obtained. Note that when the range of planning for ν/d^2 is shifted downscale this factor has the main influence, whereas the significance of diffusion is reduced. This is because the influence of the velocity of flux on the liberation of the gas component is increased, and the effect of the diffusion flux correspondingly decreased.

The dependence of the diffusion coefficient on the vibration parameters, the rheological properties of the medium, and the column height take the form

$$D = 0.438 + 0.014\,(\overline{\nu/d^2}) + 0.015\,(\bar{a}^{\text{B}}\cdot\bar{\omega}^2) + 0.182\bar{\omega} + 0.305\bar{H} - 0.005\,(\overline{\nu/d^2}\cdot\bar{H}) + 0.096\,(\bar{\omega}\bar{H}) + 0.050\bar{H}^2. \quad (10)$$

In the given case, the range of the factors being planned varied over the ranges: $\nu/d^2 = (1\text{-}2)\cdot 10^4\ \text{sec}^{-1}$; $a^U = (2\text{-}6)g\ \text{m/sec}^2$; $\omega = 20\text{-}100\ \text{sec}^{-1}$; $H = (1.5\text{-}2.5)$ m. With the given approximation, the dispersion is 0.028; the mean-square deviation is 0.040; the correlational ratio is 0.997. It follows from an analysis of Eq. (10) that the greatest influence on the diffusion coefficient comes from the column height, the angular velocity, and also the product $\bar{H}\bar{\omega}$. In addition, in the chosen range of the variable factors, the diffusion coefficient depends not only on the viscosity of the medium, but also to a large extent on the angular velocity of the vibration, the upper acceleration, and the column height.

Thus, investigation of the diffusion of the gas component in two-phase systems allows the parameters of the vibrational perturbation and the rheological parameters of the medium to be optimized for the effective reprocessing of disperse systems.

NOTATION

ρ_T, density of air bubble; V, bubble volume; v, absolute velocity of bubble motion; t, time; g, acceleration due to gravity; a, transfer acceleration; ρ, density of two-phase medium; d, diameter of air bubble; μ, dynamic viscosity; u, displacement velocity of elementary volume of medium; D, diffusion coefficient of air bubbles along coordinate x; w, bulk particle concentration of gas component; $w_i^{+\delta t}$, bulk concentration of air for the i-th point in the preceding time step ($+\delta t$); δt, time step; w_i, w_{i+1}, w_{i-1}, bulk concentrations for the given moment of time at the i-th, (i + 1)-th, and (i − 1)-th points along the x coordinate, respectively; p_{imax}, p_{imin}, maximum and minimum values, respectively, of the probability of displacement of a gas-component particle from the i-th point for the given moment of time to the i-th point for the new moment; q_{imax}, q_{imin}, maximum and minimum values, respectively, of the probability of displacement of a gas-component particle from the i-th point for the given moment to the (i + 1)-th point for the new moment; ν, kinematic viscosity.

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SEVERAL NEW OPERATIONAL CALCULUS FORMULAS

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One-dimensional Laplace transforms of some elementary and special functions are given.

The probability of obtaining a precise analytic solution to a given problem by methods of the operational calculus often depends on the presence of appropriate operation formulas in the reference tables [1, 2]. Thus, it is important to publish addenda to these tables. We note that operational methods have recently been applied to the identification of thermophysical properties [3]. The new operation formulas in Table 1 are given in double-column form. The left column contains the function f(x), and the right column, the Laplace transform F(p), where

$$F(p) = \int_0^\infty f(x) \exp(-px)\, dx$$

(Re p > 0, unless otherwise specified). The notation is standard.

TABLE 1

№	$f(x)$	$F(p)$
1	$(1-x)^n{}^n$	$\frac{1}{p^n} L_n^{-n-1}(-p)$
2	$\operatorname{ch}\sqrt{x}$	$\frac{1}{p} + \frac{1}{p^{3/2}} e^{1/(4p)} \operatorname{erf}\left(\frac{1}{2\sqrt{p}}\right)$
3	$\cos\sqrt{x}$	$\frac{1}{p} - \frac{1}{p^{3/2}} e^{-1/(4p)} \operatorname{erfi}\left(\frac{1}{2\sqrt{p}}\right)$
4	$\operatorname{ch}\sqrt{x}\cos\sqrt{x}$	$\frac{1}{p} - \frac{\sqrt{\pi}}{p^{3/2}}\left[\sin\frac{1}{2p} C\left(\frac{1}{\sqrt{2p}}\right) - \cos\frac{1}{2p} S\left(\frac{1}{\sqrt{2p}}\right)\right]$
5	$\frac{1}{x}[\operatorname{Ei}(\pm x) - \ln x - C]$	$\operatorname{Li}_2\left(\pm\frac{1}{p}\right)$ Re $p > 1$
6	$\frac{1}{x}[\operatorname{ci}(x) - \ln x - C]$	$\frac{1}{4}\operatorname{Li}_2\left(-\frac{1}{p^2}\right)$ Re $p > 1$
7	$\operatorname{Shi}(x)$	$\frac{1}{p}\operatorname{Arth} p$ Re $p > 1$
8	$\frac{1}{x}[\operatorname{chi}(x) - \ln x - C]$	$\frac{1}{4}\operatorname{Li}_2\left(\frac{1}{p^2}\right)$ Re $p > 1$

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